Development of solitons in composite right- and left-handed transmission lines periodically loaded with Schottky varactors

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The characteristics of composite right- and left-handed (CRLH) transmission lines periodically loaded with Schottky varactors are discussed in relation to the development of solitons. CRLH lines are highly dispersive and thus, when appropriately designed, compensate the effect of nonlinearity introduced by the Schottky varactors to support solitons. The reductive perturbation method applied to the transmission equation of nonlinear CRLH lines leads to the observation that the nonlinear Schrödinger equation governs the wave property at long wavelengths. The condition of the Schottky CRLH lines for the development of solitons, together with several results of the numerical finite-difference time-domain calculations, is discussed. © 2007 American Institute of Physics. [DOI: 10.1063/1.2753568]

I. INTRODUCTION

Composite right- and left-handed (CRLH) transmission lines have been intensively investigated to produce several important breakthroughs in the management of electromagnetic continuous waves. They exhibit a left-handed property, i.e., phase velocity has a sign opposite to that of the group velocity, at certain frequencies. More precisely, the wave number $k=\k(\omega)$ of the mode propagating on a CRLH line is expressed as

$$k(\omega) = \pm \sqrt{\omega^2 L_C R + \frac{1}{\omega^2 L_C L_L - \left(\frac{L_R + C_R}{C_L}\right)^2}},$$

where $L_R$, $C_L$, $L_L$, and $C_R$ are the series inductance, series capacitance, shunt inductance, and shunt capacitance, respectively. The wave number $k$ takes a positive sign at frequencies greater than $\omega_a = \max(1/\sqrt{C_L L_L}, 1/\sqrt{C_R L_L})$ and a negative sign at frequencies less than $\omega_L = \min(1/\sqrt{C_L L_R}, 1/\sqrt{C_R L_L})$ in Eq. (1). Moreover, CRLH lines have a frequency band gap, where all supporting modes are evanescent, at frequencies between $\omega_L$ and $\omega_L$.

Several investigations on the effective introduction of nonlinearity to the wave property of CRLH lines have been reported. One of the important effects of nonlinearity is the generation of solitons induced by a balance between the nonlinear and dispersive effects. It is known that the ordinary transmission line with nonlinear shunt capacitance can carry solitons. Because the dispersion of ordinary lines is weak, only the baseband solitons develop, which are described by the Toda equation. On the other hand, the CRLH line is highly dispersive; therefore, when the nonlinearity is introduced, it can carry an envelope soliton described by the nonlinear Schrödinger equation. In fiber optics, this type of soliton is commonly used for several meaningful techniques including the pulse compression or the generation of supercontinuum pulses. Moreover, the nonlinear CRLH line can support left-handed solitons. Once a design method of soliton propagation is established, the applications of CRLH lines can cover both narrow band and pulse circuits. Recently, we discussed the development of solitons in CRLH lines, with $C_R$ replaced by a varactor whose capacitance is represented by an even function of the applied voltage. Although this unfamiliar voltage dependence of varactors is realized by devices having sophisticated epitaxial structures, such as heterostructure barrier varactors, it is preferred for the development of solitons in CRLH lines with simple nonlinear devices. We thus investigate the CRLH lines periodically loaded with the Schottky varactors from the viewpoint of the development of solitons.

II. SCHOTTKY CRLH LINES

Figure 1(a) shows the circuit diagram of the observed Schottky CRLH line. The shunt capacitors are replaced by Schottky varactors, which are designed to be biased at $V_0$. The voltage dependence of Schottky varactors is given by

$$V(\phi) = \sqrt{\frac{\epsilon q N_d}{2(\phi_{bi} - V)}},$$

where $\epsilon$ and $N_d$ are the dielectric constant and the doping density of the semiconductor forming the Schottky contact, respectively. Moreover, $\phi_{bi}$ represents the built-in potential of the contact. The solid curve in Fig. 1(b) shows an example of this voltage dependence. The varactor is biased at $V_0$ and, therefore, its capacitance is $C_R^{(0)}$. For this example, we set
By introducing the spatial continuous variable $x$, replacing differences by differentials, and eliminating $I_n$ from Eqs. (6) and (7), we obtain the following single-variable equation:

$$
c_L L_R \frac{d^4 V}{dt^4} - crouch \frac{d^2 V}{dt^2} 4 \frac{d^2 V}{dx^2} + c_L \frac{d^2 V}{dt^2} + \frac{1}{2} \frac{d^2 V}{dt^2} + V = 0,
$$

where $1/c_L$, $c_R$, $1/L$, and $I_R$ are the series elastance, shunt capacitance, shunt susceptance, and series inductance per unit length, respectively. The function $V(x,t)$ is the continuous counterpart of $V_n$. Furthermore, we expand $c_R(V_0 + V)$ up to $V^2$ at approximately $V = 0$ as follows:

$$
c_R V_0 + V = c_R^{(0)} + \alpha V + \beta V^2 + O(V^3),
$$

where $c_R^{(0)} = [g e N_o / 2 (\phi_{n_0} - \phi_{V_0})]^{1/2}$, $\alpha = [g e N_o / 8 (\phi_{n_0} - \phi_{V_0})^{3/2}]$, and $\beta = [9 g e N_o / 128 (\phi_{n_0} - \phi_{V_0})^3]$. This approximated voltage dependence is shown in Fig. 1(b) by the dashed curve. It characterizes the true dependence at the voltages within $V_0 \pm 0.5$ V satisfactorily. We then apply the reductive perturbation method to Eq. (8). First, we prepare the respective spatial and temporal coordinates for the envelope and carrier waves. We use $x$ and $t$ as the spatial and temporal coordinates, respectively, for the description of the wave. For the envelope wave, $\xi = e(x - V_0 t)$ and $\tau = e t$ are used as the spatial and temporal coordinates, respectively, where $V_0$ is the velocity of the envelope wave determined in the following process. We then transform the derivatives in Eq. (8) as follows:

$$
\begin{align*}
\frac{\partial}{\partial t} &\rightarrow \frac{\partial}{\partial t} - e V_0 \frac{\partial}{\partial \xi} + e^2 \frac{\partial}{\partial \tau}, \\
\frac{\partial}{\partial x} &\rightarrow \frac{\partial}{\partial \xi} + e \frac{\partial}{\partial \tau}.
\end{align*}
$$

By this transformation, we separate the dynamics of the envelope wave from that of the carrier wave. Next, we expand the voltage variable as

$$
V = \sum_{m=1}^{\infty} \sum_{l=-\infty}^{\infty} u^{(m)}_l (\xi, \tau) e^{i(l k x - \omega t)},
$$

By substituting this into Eq. (8), $e^m$ terms are collected to make them identical zero. By solving the resulting identities, we obtain the explicit solutions of $u^{(m)}_l$. The results of reductive perturbation up to $O(\epsilon^3)$ are summarized as follows:

1. $V_0$ is equal to the group velocity of the linear wave.
2. $u^{(1)}_l = 0$ for $l \neq \pm 1$. Moreover, $u^{(1)}_{-1}$ is a complex conjugate of $u^{(1)}_1$.
3. $u^{(1)}_1$ satisfies the following nonlinear Schrödinger (NS) equation:

$$
i \frac{\partial u^{(1)}_1}{\partial \tau} + p \frac{\partial^2 u^{(1)}_1}{\partial \xi^2} + q |u^{(1)}_1|^2 u^{(1)}_1 = 0,
$$

where the coefficients $p$ and $q$ for the unbalanced Schottky CRLH lines are given by
and those for the balanced Schottky CRLH lines are given by

\[ p = \frac{c_{l}l_{w}^3}{(1 + c_{l}l_{w}^3)^3}, \]  

\[ q = -\frac{l_{w}^3(8\alpha^2\omega^2l_{w}^2(-1 + 4c_{l}l_{w}^2) - 9\beta(-5 + 8c_{l}l_{w}^2))}{6(1 + c_{l}l_{w}^3)(-5 + 8\omega c_{l}l_{w})}. \]  

It is shown that the coefficient \( p \) is one-half of GVD of the linear CRLH line resulting from the replacement of \( c_{R} \) by \( c_{R0}^{(0)} \) at long wavelengths and, therefore, \( p > 0 \) in the balanced CRLH lines and \( p > (\langle c_{R} \rangle) \) in the RH (LH) branch in the unbalanced ones, as shown by Eqs. (3)–(5). As is well known, the NS equation has soliton solutions. In particular, if \( pq > 0 \), we obtain a light soliton solution with amplitude \( A \) given by

\[ u_{1}^{(l)} = A \operatorname{sech} \left( \sqrt{\frac{q}{2p}} \right) \right \} \exp \left( \frac{qa_{l}^2 r}{2} \right). \]  

Therefore, when the contributions of \( O(\epsilon) \) components are dominant, \( V \) is calculated as

\[ V = A_{0} \operatorname{sech} \left( \sqrt{\frac{q}{8p}} A_{0}(x - V_{p} t) \right) \right \} \cos \left( \omega - \left( \omega \frac{qa_{l}^2}{8} \right) t \right). \]  

where \( A_{0} = 2eA \).

### III. NUMERICAL RESULTS

For the validation of the above-mentioned arguments, we numerically solve Eqs. (6) and (7) with \( C_{R}(V_{n}) = C_{R0}/\sqrt{\phi_{n} - V_{n} - V_{a}} \) using the standard finite difference method. We set \( C_{L} \), \( L_{R} \), \( L_{L} \), and \( \phi_{n} \) to 1.0 pF, 2.5 nH, 2.5 nH, and 0.7 V, respectively. Moreover, \( C_{R0} \) and \( V_{a} \) are set to 1.0 pF and −1.0 V, respectively. The observed Schottky CRLH line is thus balanced. It exhibits the LH property at frequencies smaller than 3.18 GHz, or it exhibits the RH property. The high-pass and low-pass cutoff frequencies are 1.32 and 7.68 GHz, respectively. The ends of the line are terminated with resistors whose resistances are matched with the long-wavelength characteristic impedance. The carrier frequency of the voltage input is set to 3.0 GHz, where the linear CRLH line exhibits the LH property. The amplitude of input is 0.3 V and the number of sections is 3000.

Figure 2 shows the results of the calculations. Seven transient waveforms are plotted, which are monitored at \( n = 1, 500, 1000, 1500, 2000, 2500, \) and 3000, respectively. No eminent distortion of the main pulse is observed, although small signals that do not contribute to the main pulse exit, which are supposed to be developed by \( u_{l}^{(m)}(m > 1) \), or radiative waves originating from the input, are observed. When \( n < 1500 \), the amplitude of the pulse decreases gradually with increasing width. After this stabilizing process, the pulse travels steadily. The main pulse at \( n = 3000 \) is shown in the inset of Fig. 2. The pulse exhibits a hyperbolic secant shape predicted by Eq. (19). For comparison, the response of the linear CRLH line to the same input pulse is shown in Fig. 3. We eliminate the voltage dependence from \( C_{R} \), such that \( C_{R}(V_{n}) = C_{R0}^{(0)} \). The other line parameters are the same as those used to obtain the results in Fig. 2. Because of the frequency shifts at the leading and trailing edges of the input, the edges travel at different group velocities from the middle part of the pulse and, therefore, the pulse is greatly broadened in the linear case. The preservation of the pulse in Fig. 2 and the similarity of the pulse shape with the analytical prediction strongly suggest the development of solitons in the Schottky CRLH lines. Note that the line is at an LH branch and, therefore, the observed soliton has an LH property.

Figure 4 shows the frequency dependence of dispersive and nonlinear coefficients. The thin solid and dashed curves show the long-wavelength dispersive and nonlinear coefficients, respectively, which are calculated by using Eqs. (16)
On the other hand, the thick solid curve shows the dispersive coefficient calculated using the exact dispersion relationship of the CRLH lines given by

\[ \cos(k) = 1 - \frac{1}{2} \left( \omega_{cl} - \frac{1}{\omega_{cL}} \right) \left( \omega_{cr} - \frac{1}{\omega_{cL}} \right). \]  

The analytical estimation is carried out using the long-wavelength approximation. However, the degree of dispersion in the numerical model is considered more precisely described by the exact dispersive coefficient. The exact dispersive coefficient is slightly greater than the long-wavelength counterpart at frequencies lower than \( f_0 = 3.18 \text{ GHz} \), which is the threshold frequency between the LH and RH branches. On the other hand, the former is less than the latter at higher frequencies than \( f_0 \) and becomes negative beyond \( f_p \). The nonlinear coefficient has two zeros at \( f_{q1}, f_{q2} \), and a pole at \( f_{q3} \), which are given by

\[ f_{q1} = \frac{5}{16 \pi c_L l_R}, \]
\[ f_{q2} = \frac{1}{4 \pi \alpha \sqrt{2 c_L l_L l_R}} \times \sqrt{\alpha^2 l_L + 9 \beta c_L l_R - \sqrt{\alpha^2 l_L^2 - 72 \alpha^2 \beta c_L l_R l_L + 81 \beta^2 c_L l_R^2}}, \]
\[ f_{q3} = \frac{1}{4 \pi \alpha \sqrt{2 c_L l_L l_R}} \times \sqrt{\alpha^2 l_L + 9 \beta c_L l_R + \sqrt{\alpha^2 l_L^2 - 72 \alpha^2 \beta c_L l_R l_L + 81 \beta^2 c_L l_R^2}}. \]

For this case, \( f_{q1}, f_{q2}, \) and \( f_{q3} \) are calculated as 2.52, 2.72, and 5.42 GHz, respectively.

As mentioned above, the light soliton can be developed only when the product of the dispersive and nonlinear coefficients becomes positive. The product becomes positive when the carrier frequency \( f \) is less than \( f_{q1} \) or \( f_{q2} < f < f_p \). At the former frequency range, the wavelength of the carrier wave is insufficient to predict the behavior of the wave by analytical estimation. At this point, the latter range, which is hatched in Fig. 4, is favorably utilized for the development of light solitons. Note that light solitons with both the RH and LH properties can be developed because of the condition \( f_{q2} < f_0 < f_p \).

### IV. CONCLUSIONS

Solitons in the Schottky CRLH lines are discussed. The Schottky varactors employed at \( C_R \) make the CRLH line a good platform for developing NS solitons. Both the LH and RH solitons are possible. Further investigations will provide a design method for developing soliton-based pulse circuits using the CRLH lines.